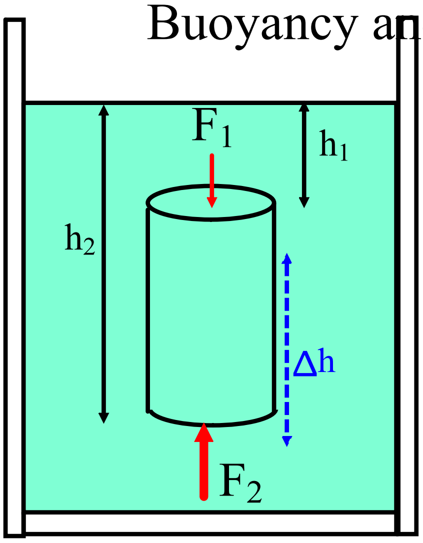
Buoyancy and Archimedes" Principle



If an object is submerged in a fluid it is buoyed (pushed up) by a force of the displaced fluid- ***it appears to weigh less because the fluid applies an upward force of the submerged object.***

This upward force is caused by the difference in pressure; less on the top of the object and greater pressure on bottom. This is due to the difference in height of top vs bottom.

$$P_{top} = \rho h_1 g$$
$$P_{bottom} = \rho h_2 g$$

$\therefore \Delta h = h_2 - h_1$ = height of object and therefore "h" of fluid displaced

$P = F/A$
$F = PA$
$F = PA$
$F = (\rho_f hg)A$
$F = \rho_f gA\Delta h$

$F_B = F_2 - F_1$
$F_B = \rho_f gA(h_2 - h_1)$
$F_B = \rho_f gA\Delta h$
$F_B = \rho_f gV \;\; = \;\; \rho_f Vg$

http://www.walter-fendt.de/ph14e/buoyforce.htm

Mr. Greschner has a mass of 92 kg an a density of 980 kg/m³.

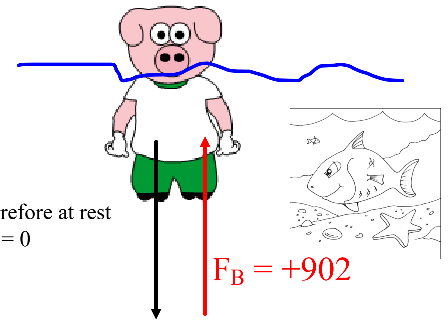
What is my volume?

$\rho = m/V$
$V = m/\rho$
$V = 92\text{ kg}/ 980\text{ kg/m}^3$
$V = .094\text{ m}^3$

What is my weight?

$F_w = F_o$
$F_o = mg = 92\text{ kg}(-9.8\text{ m/s}^2)$
$F_o = -902\text{ N}$

Mr. G swims with the fishes!

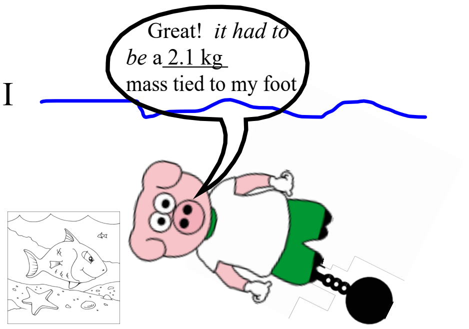


I float and am therefore at rest
$\Sigma F = 0 = F_w + F_b = 0$
$F_b = -F_w$

$F_o = -902\text{ N}$

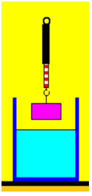
What buoyancy does the water apply to me if I dive(not voluntarily) under the water?

$F_B = \rho Vg$
$F_B = 1000\text{ kg/m}^3(.094\text{ m}^3)g = +922\text{ N}$

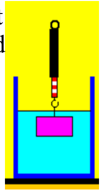


$$\underset{\text{weight of fluid}}{\underset{\nearrow}{F_B}} = \underset{\text{in air}}{\underset{\uparrow}{F_o}} - \underset{\text{in fluid}}{\underset{\uparrow}{F_{app}}}$$

F_o = weight in air



F_{app} = weight in fluid



invert & multiply

if sinks: $V_o = V_f$

V's cancel

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o} \therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

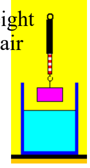
$$S.G. = \frac{F_o}{F_B} = \frac{F_{o\ in\ air}}{F_{o\ in\ air} - F_{app\ in\ water}} = \frac{F_{o\ in\ air}}{\Delta F}$$

$$F_B = \underset{\text{in air}}{\underset{\uparrow}{F_o}} - \underset{\text{in fluid}}{\underset{\uparrow}{F_{app}}}$$

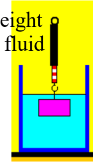
or, $F_{app} = F_o - F_B$

$$\underset{\text{weight of fluid}}{\underset{\nearrow}{F_B}} = \underset{\text{in air}}{\underset{\uparrow}{F_o}} - \underset{\text{in fluid}}{\underset{\uparrow}{F_{app}}}$$

F_o = weight in air



F_{app} = weight in fluid



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$$S.G. = \frac{F_o}{F_B} = \frac{F_{o\ in\ air}}{F_{o\ in\ air} - F_{app\ in\ water}} = \frac{F_{o\ in\ air}}{\Delta F}$$

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if floats, $m_o = m_f \approx F_o = F_B$

invert & multiply

m's cancel

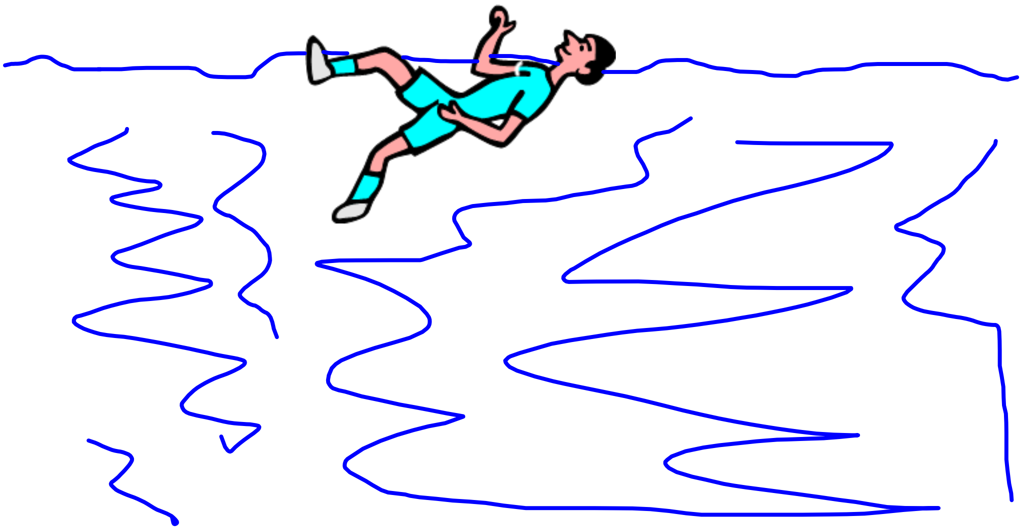
$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o} \therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$$

(% submerged)

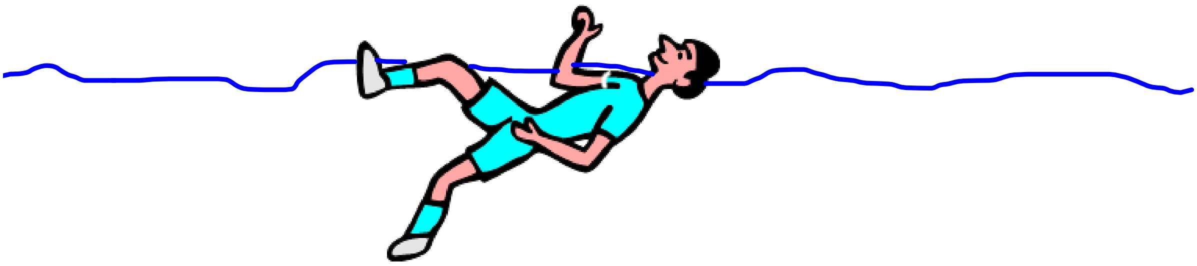
An 88.0 kg runner falls in the lake.

$\rho_o = 755\ \text{kg/m}^3$

Oh no!



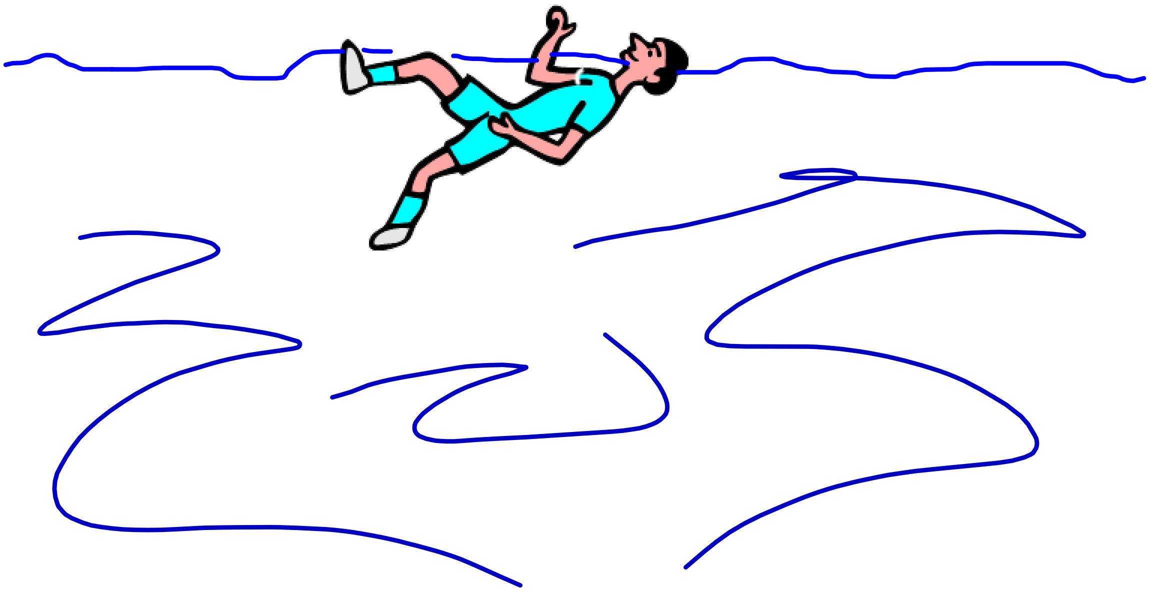
What is the buoyancy force applied to him/her?



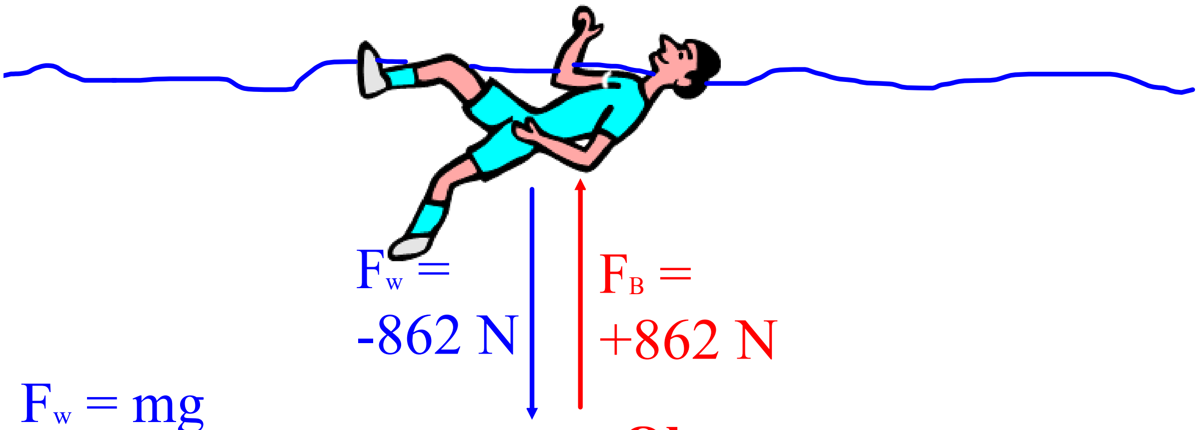
Observe:

"rest" – therefore, $\Sigma F = 0$

What is the buoyancy force applied to him/her? .... m = 88 kg



What is the buoyancy force applied to him/her?



$F_w =$ -862 N

$F_B =$ +862 N

$F_w = mg$

$F_w = 88.0 \text{ kg} \times 9.8 \text{m/s}^2$

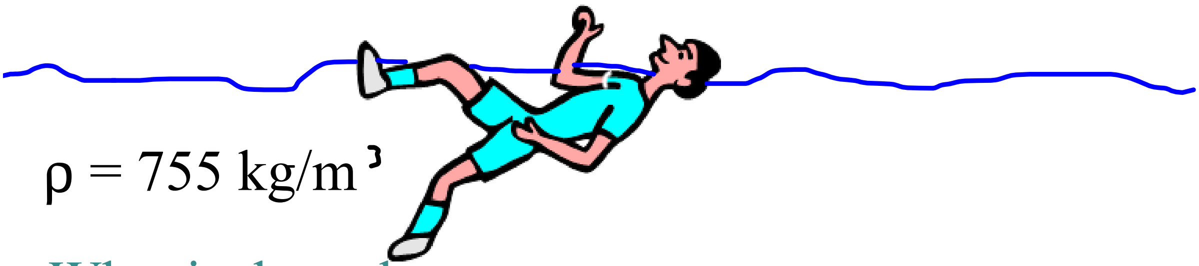
$F_w = -862 \text{ N}$

Observe:

"rest" – therefore, $\Sigma F = 0$

$F_w + F_B = 0$

$F_B = +862 \text{ N}$



$\rho = 755\ kg/m^3$

What is the volume of the runner?

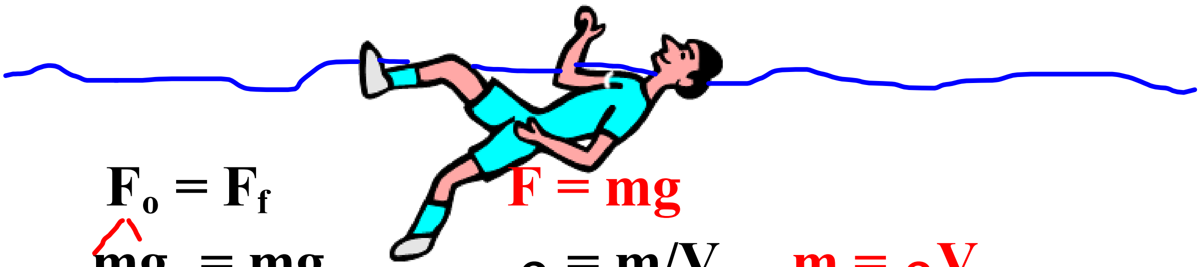
$\rho = m/V$

$V_o = m/\rho$

$V_o = 88.0\ kg/755\ kg/m^3$

$V_o = .117\ m^3$

What is the volume of water displaced by the runner?



$F_o = F_f$ $\quad$ $F = mg$

$mg_o = mg_f$ $\quad$ $\rho = m/V$ ....$m = \rho V$

$\rho V g_o = \rho V g_F$ $\quad$ g's cancel

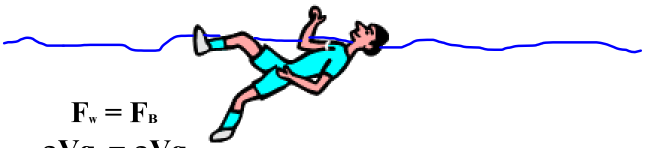
$\rho V_o = \rho V_F$

$V_F = \rho V_o/\rho_F$

$V_F = (755\ kg\ m^3\ 0.117m^3)/1000kg/m^3$

$V_F = .088\ m^3$

What is the volume of water displaced by the runner?



$F_w = F_B$

$\rho V g_o = \rho V g_F$

$\rho V_o = \rho V_F$

$V_F = \rho V_o / \rho_F$

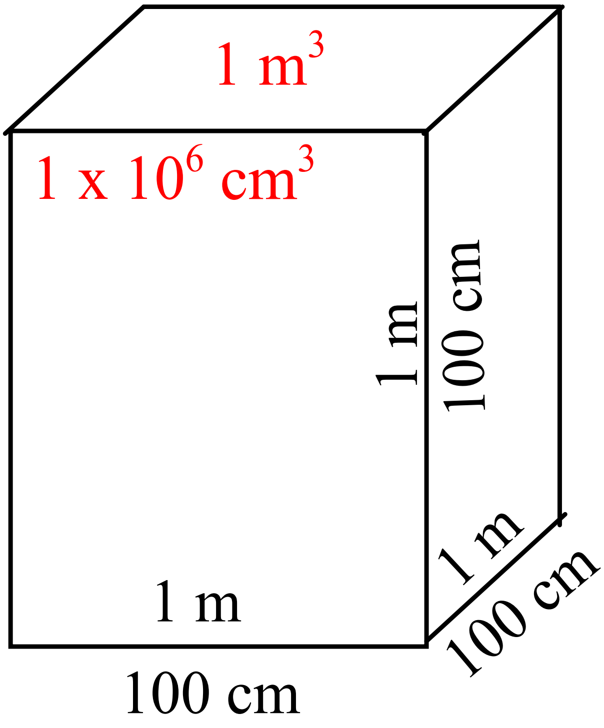
$V_F = (755 \text{ kg m}^3 \, 0.117\text{m})^3/1000\text{kg/m}^3$

$V_F = .088 \text{ m}^3$

What is the mass of the 0.088 m^3 of water?

1 *liter* of water has a mass of 1kg, and there are 1000 *liters* in 1 m^3 of water. *Therefore,* .088 m^3 of water is 88 *liters, is 88 kg!!!*

or, ρ = **m/V ....m = ρV = 1000kg/m^3(.088 m^3) = 88 kg**



Fluid (water)

$V = .088\ m^3$

$\rho = m/V$

$m = \rho V$

$m = 1000\ kg/m^3(.088\ m^3)$

$m = 88\ kg$

An Egyptian soldier has a mass of 125 kg and a volume of 0.117 m when he's fitted for battle. March Egyptian soldier, march!

The marching Egyptian soldier falling into the lake (Sea) and begins to sink in the water.

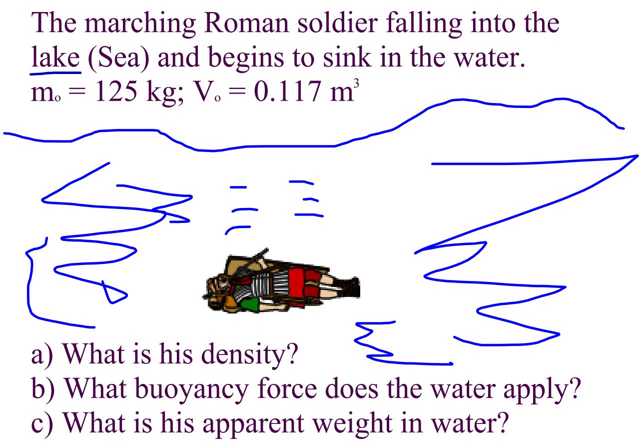
$m_o = 125$ kg; $V_o = 0.117$ m^3

Sink Egyptian soldier, sink

a) What is his density?

b) What buoyancy force does the water apply?

c) What is his apparent weight in water?



The marching Roman soldier falling into the lake (Sea) and begins to sink in the water.

$m_o = 125$ kg; $V_o = 0.117$ m^3

a) What is his density?

b) What buoyancy force does the water apply?

c) What is his apparent weight in water?

Apparent weight is what you *appear* to weigh when you're submerged in a fluid. Gravity pushed down on you (that's your F_w) and the fluid pushes up (that's the buoyancy force, F_b). Your apparent weight is the difference of the two. $F_{app} = F_w - F_b$ (or, the sum of the two if you use vector directions)

$\rho_o = ?$

$\rho_o = m/V = 125\ kg/0.117\ m^3 = 1070\ kg/m^3$

$F_B = ?$

sink or float?

$V_o = V_f$ $V_0 > V_f$

$F_B = mg_f = \rho V g_f$

$F_B = \rho V g_f$

$F_B = 1000\ kg/m^3(0.117\ m^3)9.81\ m/s^2 = 1150\ kg\ m/s^2$

1150 N

his weight is: $F_w = mg = 125\ kg(9.81\ m/s^2) = 1226\ N$

$F_{app} = ?$

$F_w = F_{app} + F_B$

$F_{app} = F_w - F_B$

$F_{app} = mg - F_B$

$F_{app} = 125\ kg(9.81\ m/s^2) - 1150\ N$

$F_{app} = 76\ N$

Specific Gravity: ratio of an objects density to that of water

floats

$m_o = m_f$ $(F_o = F_B)$

$\therefore$ m's cancel

$$\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$$

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

$$\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

sinks

$V_o = V_f$

$\therefore$ V's cancel

$$F_B = \underset{\substack{\uparrow \\ \text{in air}}}{F_o} - \underset{\substack{\uparrow \\ \text{in fluid}}}{F_{app}}$$

$$F_B = \underset{\substack{\uparrow \\ \text{in air}}}{F_o} - \underset{\substack{\uparrow \\ \text{in fluid}}}{F_{app}}$$

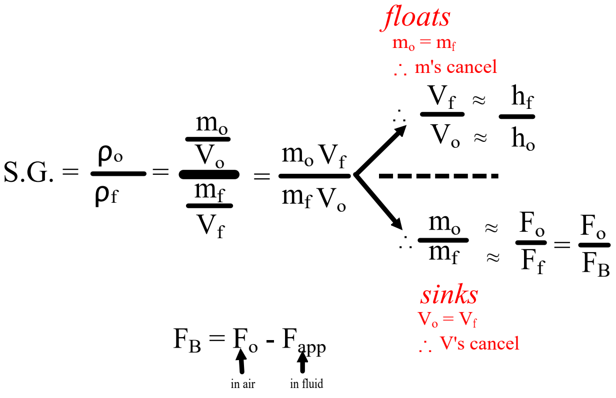
if sinks $V_o = V_f$

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o} \therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

$$S.G. = \frac{F_o}{F_B} = \frac{F_{o\ \textit{in air}}}{F_{o\ \textit{in air}} - F_{app\ \textit{in water}}} = \frac{F_{o\ \textit{in air}}}{\Delta F}$$

(either vector sum or magnitude difference!)

Specific Gravity: ratio of an objects density to that of water



floats

$m_o = m_f$

$\therefore$ m's cancel

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

$$\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$$

$$\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

sinks

$V_o = V_f$

$\therefore$ V's cancel

$$F_B = \underset{\substack{\uparrow \\ \text{in air}}}{F_o} - \underset{\substack{\uparrow \\ \text{in fluid}}}{F_{app}}$$

Egyptian soldier sinks. His density is 1070 kg/m^3

$$S.G. = \rho_o/\rho_f = \frac{1070\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = 1.07$$

$$S.G. = m_o/m_f = 125\ \text{kg}/117\ \text{kg} = 1.07$$

Specific Gravity: ratio of an objects density to that of water

$$\text{S.G.} = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

floats
$m_o = m_f$
∴ m's cancel

$$\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$$

$$\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

sinks
$V_o = V_f$
∴ V's cancel

$$F_B = \underset{\text{in air}}{\underset{\uparrow}{F_o}} - \underset{\text{in fluid}}{\underset{\uparrow}{F_{app}}}$$

The runner floats: Its density is 755 kg/m^3

$$\text{S.G.} = \rho_o/\rho_f = \frac{755\text{ kg/m}^3}{1000\text{ kg/m}^3} = .755$$

75.5% under water

$$\text{S.G.} = V_f/V_o = .088\text{ m}^3/.117\text{ m}^3 = .752$$

Specific Gravity: ratio of an objects density to that of water

$$\text{S.G.} = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

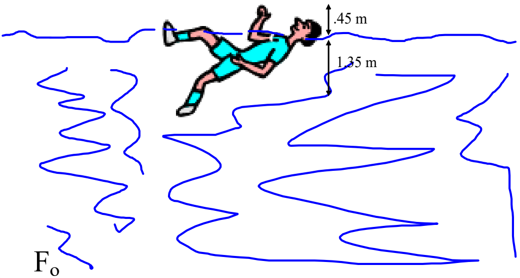
floats
$m_o = m_f$
∴ m's cancel

$$\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$$

$$\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

sinks
$V_o = V_f$
∴ V's cancel

$$F_B = \underset{\text{in air}}{\underset{\uparrow}{F_o}} - \underset{\text{in fluid}}{\underset{\uparrow}{F_{app}}}$$



Herm is 1.8 m tall. .45 m of it is above the water and 1.35 m below!

The runner floats: Her/Him is 1.8 m tall.

$$\text{S.G.} = V_f/V_o = .088\text{ m}^3/.117\text{ m}^3 = .752$$

$$\text{S.G.} = h_f/h_o \quad h_f = \text{S.G. } h_o = .752\ (1.8\text{ m}) = 1.35\text{ m}$$

1.35 m of Her is under water and .45 m is above.
(1.8 m - 1.35 m = .45 m)

http://www.walter-fendt.de/ph11e/buoyforce.htm