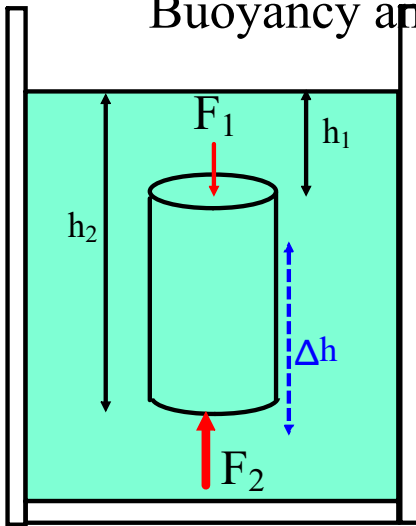


Buoyancy and Archimedes' Principle



If an object is submerged in a fluid it is buoyed (pushed up) by a force of the displaced fluid- *it appears to weigh less because the fluid applies an upward force of the submerged object.*

This upward force is caused by the **difference in pressure**; less on the top of the object and greater pressure on bottom. This is due to the difference in height of top vs bottom.

$$P_{\text{top}} = \rho h_1 g$$

$$P_{\text{bottom}} = \rho h_2 g$$

$\therefore \Delta h = h_2 - h_1 =$ height of object and therefore "h" of fluid displaced

$$P = F/A$$

$$F = PA$$

$$F = \rho h g A$$

$$F_B = F_2 - F_1$$

$$F_B = \rho_f g A (h_2 - h_1)$$

$$F_B = \rho_f g A \Delta h$$

$$A \Delta h = V$$

$$F_B = \rho_f g V = \rho_f V g$$

<http://www.walter-fendt.de/ph14e/buoyforce.htm>



Mr. Greschner has a mass of 92 kg and a density of 980 kg/m³.

What is my volume?

$$\rho = m/V$$

$$V = m/\rho$$

$$V = 92 \text{ kg} / 980 \text{ kg/m}^3$$

$$V = .094 \text{ m}^3$$

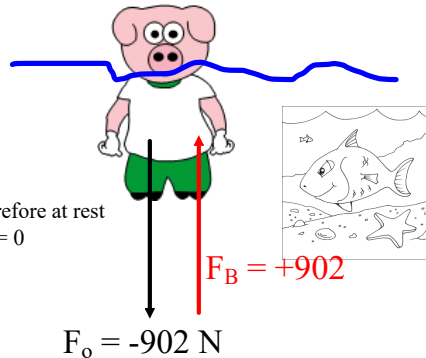
What is my weight?

$$F_w = F_o$$

$$F_o = mg = 92 \text{ kg} (-9.8 \text{ m/s}^2)$$

$$F_o = -902 \text{ N}$$

Mr. G swims with the fishes!

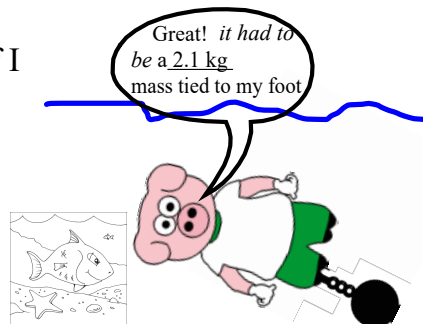


I float and am therefore at rest
 $\Sigma F = 0 = F_w + F_b = 0$
 $F_b = -F_w$

What buoyancy does the water apply to me if I dive (not voluntarily) under the water?

$$F_B = \rho V g$$

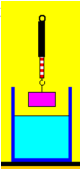
$$F_B = 1000 \text{ kg/m}^3 (.094 \text{ m}^3) g = +922 \text{ N}$$



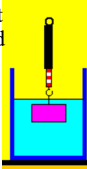
$$F_B = F_o - F_{app}$$

\nearrow weight of fluid
 \uparrow in air
 \uparrow in fluid

$F_o = \text{weight in air}$



$F_{app} = \text{weight in fluid}$



$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} \stackrel{\text{invert \& multiply}}{=} \frac{m_o V_f}{m_f V_o} \stackrel{\text{if sinks: } V_o = V_f \text{ V's cancel}}{\therefore} \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

$$S.G. = \frac{F_o}{F_B} = \frac{F_o \text{ in air}}{F_o \text{ in air} - F_{app} \text{ in water}} = \frac{F_o \text{ in air}}{\Delta F}$$

$$F_B = F_o - F_{app} \quad \text{or, } F_{app} = F_o - F_B$$

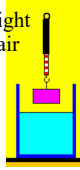
\uparrow in air
 \uparrow in fluid

[Specific Gravity (S.G.) is the density of an object compared to water]

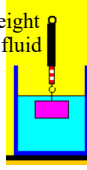
$$F_B = F_o - F_{app}$$

\nearrow weight of fluid
 \uparrow in air
 \uparrow in fluid

$F_o = \text{weight in air}$



$F_{app} = \text{weight in fluid}$



$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} \stackrel{\text{invert \& multiply}}{=} \frac{m_o V_f}{m_f V_o} \stackrel{\text{if sinks: } V_o = V_f \text{ V's cancel}}{\therefore} \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

$$S.G. = \frac{F_o}{F_B} = \frac{F_o \text{ in air}}{F_o \text{ in air} - F_{app} \text{ in water}} = \frac{F_o \text{ in air}}{\Delta F}$$

if floats, $m_o = m_f \approx F_o = F_B$

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} \stackrel{\text{invert \& multiply}}{=} \frac{m_o V_f}{m_f V_o} \stackrel{\text{m's cancel}}{\therefore} \frac{V_f}{V_o} \approx \frac{h_f}{h_o} \quad (\% \text{ submerged})$$



An 88.0 kg runner falls in the lake.

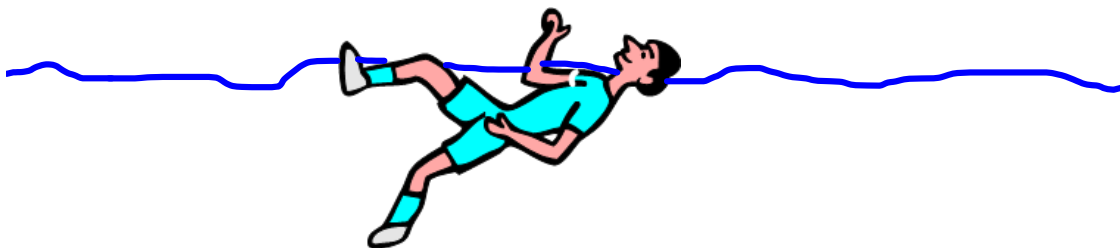
$$\rho_o = 755 \text{ kg/m}^3$$



Oh no!



What is the buoyancy force applied to him/her?



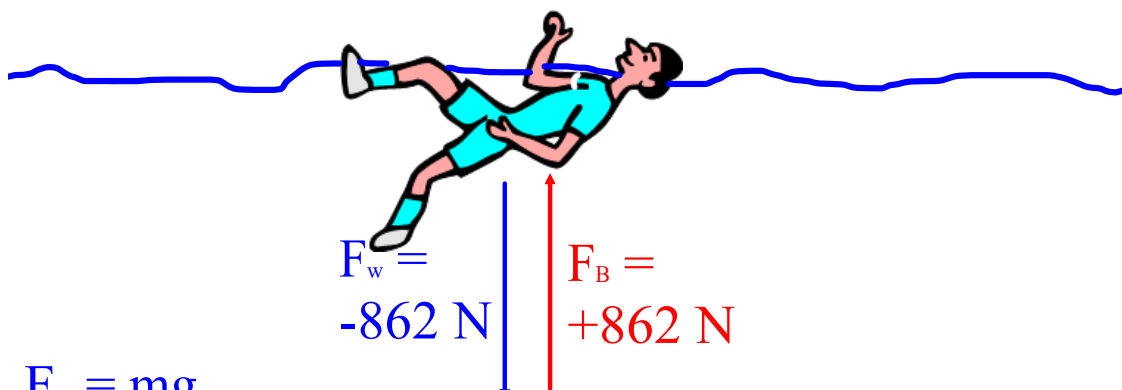
Observe:

"rest" – therefore, $\Sigma F = 0$

What is the buoyancy force applied to him/her? $m = 88 \text{ kg}$



What is the buoyancy force applied to him/her?



$$F_w = mg$$

$$F_w = 88.0 \text{ kg} \times 9.8 \text{ m/s}^2$$

$$F_w = -862 \text{ N}$$

Observe:

"rest" – therefore, $\Sigma F = 0$

$$F_w + F_B = 0$$

$$F_B = +862 \text{ N}$$



$$\rho = 755 \text{ kg/m}^3$$

What is the volume
of the runner?

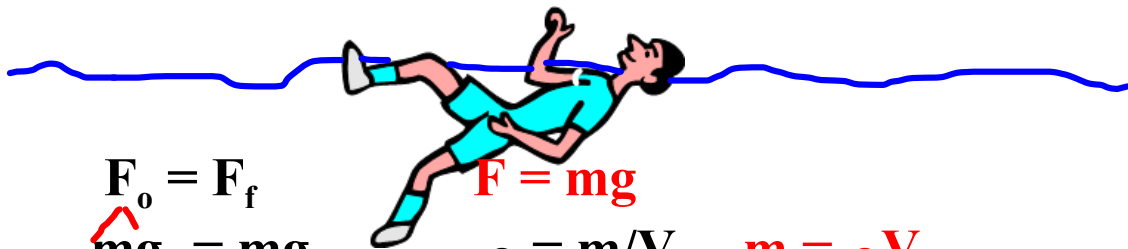
$$\rho = m/V$$

$$V_o = m/\rho$$

$$V_o = 88.0 \text{ kg}/755 \text{ kg/m}^3$$

$$V_o = .117 \text{ m}^3$$

What is the volume of water displaced by the
runner?



$$F_o = F_f$$

$$mg_o = mg_f$$

$$\rho V g_o = \rho V g_f$$

$$\rho V_o = \rho V_f$$

$$V_f = \rho V_o / \rho_f$$

$$V_f = (755 \text{ kg m}^3 \cdot 0.117 \text{ m}^3) / 1000 \text{ kg/m}^3$$

$$V_f = .088 \text{ m}^3$$

$$F = mg$$

$$\rho = m/V \dots m = \rho V$$

g's cancel

What is the volume of water displaced by the runner?



$$F_w = F_B$$

$$\rho V g_o = \rho V g_f$$

$$\rho V_o = \rho V_f$$

$$V_f = \rho V_o / \rho_f$$

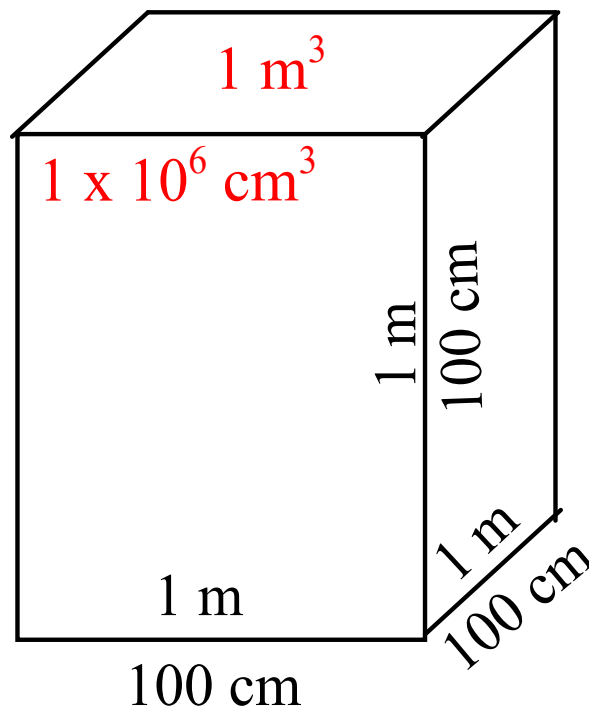
$$V_f = (755 \text{ kg} \cdot 0.117 \text{ m}) / 1000 \text{ kg/m}^3$$

$$V_f = .088 \text{ m}^3$$

What is the mass of the 0.088 m^3 of water?

1 *liter* of water has a mass of 1kg, and there are 1000 *liters* in 1 m^3 of water. Therefore, $.088 \text{ m}^3$ of water is 88 *liters*, is 88 kg!!!

or, $\rho = m/V \dots m = \rho V = 1000 \text{ kg/m}^3 (.088 \text{ m}^3) = 88 \text{ kg}$



Fluid (water)

$$V = .088 \text{ m}^3$$

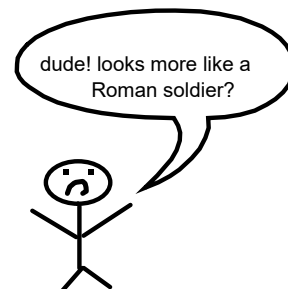
$$\rho = m/V$$

$$m = \rho V$$

$$m = 1000 \text{ kg/m}^3 (.088 \text{ m}^3)$$

$$m = 88 \text{ kg}$$

An Egyptian soldier has a mass of 125 kg and a volume of 0.117 m when he's fitted for battle. March Egyptian soldier, march!



The marching Egyptian soldier falling into the lake (Sea) and begins to sink in the water.

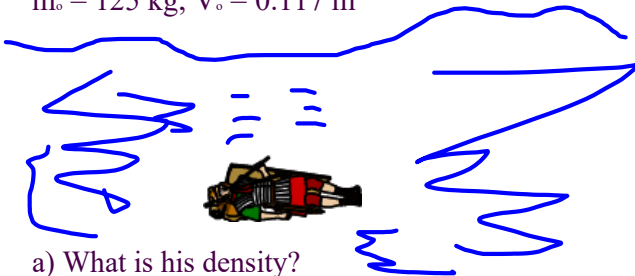
$$m_o = 125 \text{ kg}; V_o = 0.117 \text{ m}^3 \quad \text{Sink Egyptian soldier, sink}$$



- What is his density?
- What buoyancy force does the water apply?
- What is his apparent weight in water?

The marching Roman soldier falling into the lake (Sea) and begins to sink in the water.

$$m_o = 125 \text{ kg}; V_o = 0.117 \text{ m}^3$$



- What is his density?
- What buoyancy force does the water apply?
- What is his apparent weight in water?

Apparent weight is what you *appear* to weigh when you're submerged in a fluid. Gravity pushed down on you (that's your F_w) and the fluid pushes up (that's the buoyancy force, F_b). Your apparent weight is the difference of the two. $F_{app} = F_w - F_b$ (or, the sum of the two if you use vector directions)

$$\rho_o = ?$$

$$\rho_o = m/V = 125 \text{ kg}/0.117 \text{ m}^3 = 1070 \text{ kg/m}^3$$

$$F_B = ? \quad F_B = mg_f = \rho V g_f$$

$$F_B = \rho V g_f$$

sink or float?

$V_o = V_f$ $V_o > V_f$

$$F_B = 1000 \text{ kg/m}^3 (0.117 \text{ m}^3) 9.81 \text{ m/s}^2 = 1150 \text{ kg m/s}^2$$

$$1150 \text{ N}$$

his weight is: $F_w = mg = 125 \text{ kg}(9.81 \text{ m/s}^2) = 1226 \text{ N}$

$$F_{app} = ? \quad F_w = F_{app} + F_B$$

$$F_{app} = F_w - F_B$$

$$F_{app} = mg - F_B$$

$$F_{app} = 125 \text{ kg}(9.81 \text{ m/s}^2) - 1150 \text{ N}$$

$$F_{app} = 76 \text{ N}$$

Specific Gravity: ratio of an objects density to that of water

$$\text{S.G.} = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

floats

$m_o = m_f \quad (F_o = F_B)$

$\therefore m\text{'s cancel}$

$\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$

sinks

$V_o = V_f$

$\therefore V\text{'s cancel}$

$$F_B = F_o - F_{app}$$

\uparrow
in air

\uparrow
in fluid

$$F_B = F_o - F_{app}$$

\uparrow \uparrow
 in air in fluid

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o} \quad \therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$$

if sinks $V_o = V_f$

$$S.G. = \frac{F_o}{F_B} = \frac{F_o \text{ in air}}{F_o \text{ in air} - F_{app} \text{ in water}} = \frac{F_o \text{ in air}}{\Delta F}$$

(either vector sum or magnitude difference!)

Specific Gravity: ratio of an objects density to that of water

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

floats
 $m_o = m_f$
 \therefore m's cancel
 $\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$

$\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$
sinks
 $V_o = V_f$
 \therefore V's cancel

$F_B = F_o - F_{app}$
 \uparrow \uparrow
 in air in fluid

Egyptian soldier sinks. His density is 1070 kg/m³

$$S.G. = \rho_o / \rho_f = \frac{1070 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 1.07$$

$$S.G. = m_o / m_f = \frac{125 \text{ kg}}{117 \text{ kg}} = 1.07$$

Specific Gravity: ratio of an objects density to that of water

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

floats
 $m_o = m_f$
 $\therefore m\text{'s cancel}$
 $\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$

sinks
 $V_o = V_f$
 $\therefore V\text{'s cancel}$
 $\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$

$F_B = \underset{\substack{\uparrow \\ \text{in air}}}{F_o} - \underset{\substack{\uparrow \\ \text{in fluid}}}{F_{app}}$



The runner floats: Its density is 755 kg/m³

$$S.G. = \rho_o / \rho_f = \frac{755 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = .755$$

75.5% under water

$$S.G. = V_f / V_o = .088 \text{ m}^3 / .117 \text{ m}^3 = .752$$

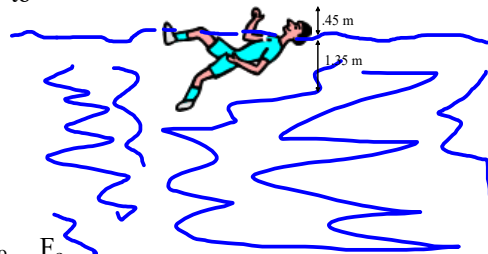
Specific Gravity: ratio of an objects density to that of water

$$S.G. = \frac{\rho_o}{\rho_f} = \frac{\frac{m_o}{V_o}}{\frac{m_f}{V_f}} = \frac{m_o V_f}{m_f V_o}$$

floats
 $m_o = m_f$
 $\therefore m\text{'s cancel}$
 $\therefore \frac{V_f}{V_o} \approx \frac{h_f}{h_o}$

sinks
 $V_o = V_f$
 $\therefore V\text{'s cancel}$
 $\therefore \frac{m_o}{m_f} \approx \frac{F_o}{F_f} = \frac{F_o}{F_B}$

$F_B = \underset{\substack{\uparrow \\ \text{in air}}}{F_o} - \underset{\substack{\uparrow \\ \text{in fluid}}}{F_{app}}$



Herm is 1.8 m tall. .45 m of it is above the water and 1.35 m below!

The runner floats: Her/Him is 1.8 m tall.

$$S.G. = V_f / V_o = .088 \text{ m}^3 / .117 \text{ m}^3 = .752$$

$$S.G. = h_f / h_o \quad h_f = S.G. \cdot h_o = .752 (1.8 \text{ m}) = 1.35 \text{ m}$$

1.35 m of Her is under water and .45 m is above.

$$(1.8 \text{ m} - 1.35 \text{ m} = .45 \text{ m})$$

<http://www.walter-fendt.de/ph11e/buoyforce.htm>

